Zero-knowledge and multiparty computations

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1 More counter-intuitive things
- Secure multi-party computation
- Zero-knowledge proofs of knowledge

Example (Yao's Millionaires' Problem)

- Two millionaires meet in the street.
- They want to find out who is the richer.
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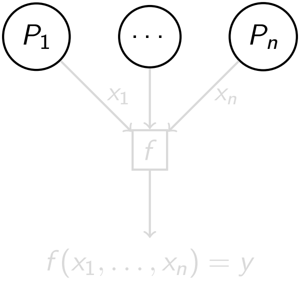
- We have n participants $P_1, \ldots, P_n$.
- Each person has a *secret* input value x_j for $1 \leq j \leq n$.
- But they desperately want to know $y = f(x_1, \ldots, x_n)$.



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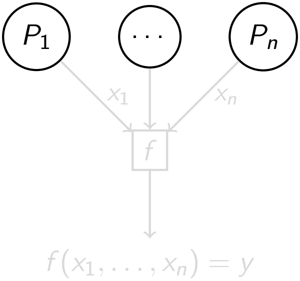

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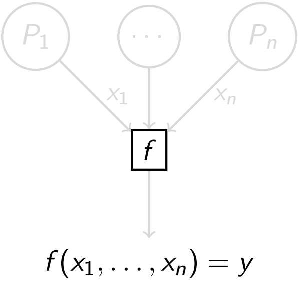

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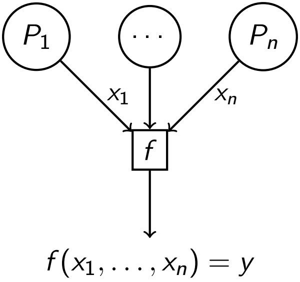

Example (Trivial solution)

- The n participants $P_1, \ldots, P_n$ agree on a trusted third-party (TTP).
- Each participant give their secret to the TTP.
- The TTP trusted third-party performs the computation.
- Every participant receives the result from the TTP.

Definition (Secure multiparty computation, MPC)

- n participants $P_1, \ldots, P_n$.
- n secret inputs $x_1, \ldots, x_n$.
- A protocol π is executed by the participants.
- At the end of the protocol each participant learns $y = f(x_1, \ldots, x_n)$.
- The participants executing π should be *equivalent* to giving $x_1, \ldots, x_n$ to a TTP T who computes $f(x_1, \ldots, x_n) = y$ and returns y to each participant.

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Each participant P_i learns no more about x_j $(i \neq j)$ than what is revealed by y.

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- We can construct protocols for arbitrary functions f.
- Efficiency varies though.
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- Several thousand farmers produce sugar beets.
- These are sold to the monopoly Danisco, the sugar producer.
- Contracts are allocated via a nation-wide exchange, a double auction.
- A double auction contains multiple sellers and multiple buyers.
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- Each buyer places a bid specifying how much he is willing to buy *at each potential price*.
- Each seller says how much they are willing to sell at each given price.
- The auctioneer computes the total supply and demand for each price.
- We want to find where supply equals demand.
- When done, anyone who specified non-zero for this price may trade at this price.

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Example

- Alice must prove her identity to Eve.
- Eve has Alice's public key, and knows it belongs to Alice.
- Alice wants to prove she is the owner of the private key belonging to the public key that Eve has.
- Eve asks Alice to sign the message m, if the signature verifies under the public key Eve believes Alice.

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- Now Eve can show this message (chosen by Eve) with Alice's signature on it!
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Idea

- Alice wants to prove that she knows the discrete logarithm x of a value g^x.
- She will do this without revealing x to Eve.

Definition (Schnorr's protocol[2])

- Prover wants to prove knowledge of x for $g^x = y$.
- Prover commits to randomness r, by sending $t = g^r$.
- Verifier replies with randomly chosen challenge c.
- After receiving c, prover replies with $s = r + cx$.
- Verifier accepts if $g^s = g^{r+cx} = g^r(g^x)^c = ty^c$.

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Proof outline.

- We need to prove *completeness*: for all (most) statements the verifier will accept.
- We need to prove *soundness*: for all (most) false statements the verifier will reject.
- We need to prove that it is zero-knowledge.

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Zero-knowledge

- Transcript for protocol: (t, c, s).
- Probability for transcript occurring: $\frac{1}{|R|} \cdot \frac{1}{|R'|}$.
- Simulate protocol: randomly choose c, randomly choose s, compute t by $g^s y^c$.
- We see that we get the same probability distribution.
- Thus the simulated transcripts are indistinguishable from the real ones.

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Zero-knowledge proofs of knowledge